LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **PHYSICS**

FIRST SEMESTER – NOVEMBER 2011

PH 1814/PH 1809 - CLASSICAL MECHANICS

Date : 05-11-2011 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

PART – A

Answer **ALL** the questions (10 X 2 = 20)

01. State any two differences between the Lagrangian and the Hamiltonian.
02. If $L = ½ m(\dot{r}^2 + r^2\dot{\theta}^2) – V(r)$ and θ is a cyclic coordinate. Find $\mathbf{p}_\theta$.
03. Prove that d/dt[mT] = **F.p** where T is the kinetic energy of the particle.
04. Write the Euler's equations for the symmetric top with moments of inertia I_1, I_2, and I_3.
05. State the canonical equations of motion for the Hamiltonian H of a system of particles.
06. Show that the Hamiltonian is a constant of motion if it is not an explicit function of time.
07. Show that the generating function $F_3 = pQ$ generates an identity transformation with a negative sign.
08. Show that $[p_x,L_z] = -p_y$
09. State Jacobi identity.
10. Define Hamilton's principal function S

PART – B

Answer any **FOUR** questions (4 X 7.5 =30)

11. Set up the Lagrangian for a particle of mass m in a central force using polar coordinates (r, θ) and hence obtain the differential equation of orbit of the form: $d^2u/d\theta^2 + u = -m/l^2\, d/du[V(1/u)]$.
12. Obtain the Euler's equations of motion for a rigid body acted upon by a torque **N.**
13. Derive an expression for the Coriolis force and state any one example as an illustration of the Coriolis force.
14. Show that the transformation Q = q + ip and P = q – ip is not canonical. Suppose the size of the units used to measure the coordinates and momenta are changed to Q' and P' such that Q' = μQ and P' = νP then show the transformation equations are canonical if μ = i/2ν
15. Define action and angle variables. Using the action angle variable method show that the frequency of the one dimensional oscillator is $\nu = (1/2\pi)\sqrt{(k/m)}$ (3 + 4.5)

PART – C

Answer any **FOUR** questions (4 X 12.5 =50)

16. a) Obtain the Lagrange's equations from the variational principle for a holonomic system.
    b) Using the definition of the Hamiltonian $H = \Sigma p_i\dot{q}_i – L$ prove that the Hamiltonian is a sum of kinetic energy and potential energy. (7.5 + 5)
17. a) A particle of mass m is attached to the mid-point of a weightless rod of length L. The ends of the rod are constrained to move along the x and y axes without friction. Write the Lagrangian and solve the equation of motion assuming that gravitational field acts in the negative y direction. (6)
    b) A particle of mass m moves in one dimension such that it has the Lagrangian $L = m^2\dot{x}^4/12 + m\dot{x}^2/4V(x) – V^2(x)$ where V is some differentiable function of x. Find the equation of motion for x. (6.5)
18. a) Obtain the transformation equations for the generating functions $F_2(q,P,t)$ and $F_3(p,Q,t)$
    b) Show that the transformation given by $2P = p^2+q^2$ and $Q = \tan^{-1}q/p$ is canonical. (8 + 4.5)
19. a) Solve by the Hamilton-Jacobi method the motion of a particle in a plane under the action of a central potential V(r) to obtain the equation of orbit. (7.5)
    b) Solve the motion of a particle in one dimension whose Hamiltonian is given by
    $H = p^2/2m + V(q)$. (5)
20. Write notes on any TWO of the following
    i) Rutherford scattering formula ii) Fundamental Poisson's brackets.
    iii) Kepler's third law by action-angle variable method.

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